Aligning the community college math curriculum with the Common Core State Standards in Math

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What are the Common Core State Standards?

"Educational standards [that] describe what students should know and be able to do in each subject in each grade. In California, the State Board of Education decides on the standards for all students, from kindergarten through high school."
http://www.cde.ca.gov/re/cc/tl/whatareccss.asp

What are the Common Core State Standards? (continued)

Forty-four states, the District of Columbia, four territories, and the Department of Defense Education Activity have adopted the Common Core State Standards.
http://www.corestandards.org/standards-in-your-state/

What are the Common Core State Standards? (continued)

California is part of the Smarter Balanced Assessment Consortium (SBAC). SBAC uses computer-based, adaptive testing.

The other major CCSS assessment consortium is the Partnership for Assessment of Readiness for College and Careers (PARCC)

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Not only will our incoming students be differently prepared, but our articulation and transfer agreements with four-year schools may be altered by the changed expectations of what it means to be "college and career ready."

BOARS clarified (December 2013) that "… going forward, all students must complete the basic mathematics defined by the college-ready standards of the Common Core State Standards for Mathematics (CCSSM) prior to enrolling in a UC-transferable college mathematics or statistics course." [Emphasis mine]

How are CA community colleges affected by the CCSS in Math? (continued)

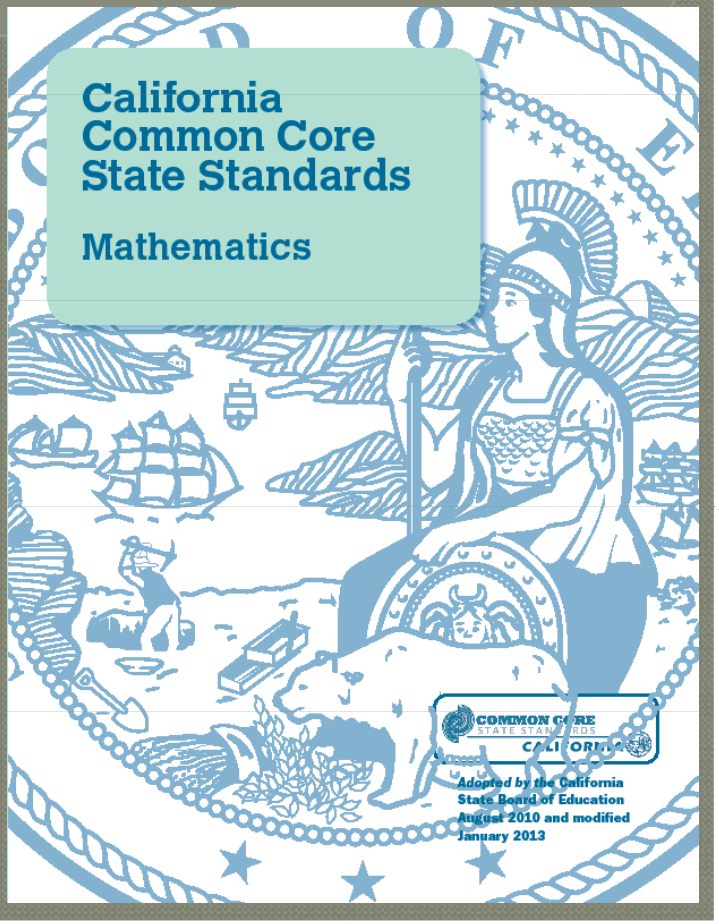
Our assessment and placement instruments will need adjustment or replacement.

How are CA community colleges affected by the CCSS in Math? (continued)

Our assessment and placement instruments will need adjustment or replacement. (The existing CA Early Assessment Program exempts students who meet a set score on the 11th grade assessment from taking placement exams in CCCs and certifies that these students are ready for transfer-level math courses.)

So what's in the CCSSM?

- Standards for Mathematical Practice
- Standards for Mathematical Content



8 Standards for Mathematical Practice

The MP describe how math students ought to engage with the subject matter as they grow in mathematical maturity and expertise throughout the elementary, middle ,and high school years.

8 Standards for Mathematical Practice

1) Make sense of problems and persevere in solving them.

8 Standards for Mathematical Practice

2) Reason abstractly and quantitatively.

8 Standards for Mathematical Practice

3) Construct viable arguments and critique the reasoning of others.

8 Standards for Mathematical Practice

4) Model with mathematics.

8 Standards for Mathematical Practice

5) Use appropriate tools strategically.

8 Standards for Mathematical Practice

6) Attend to precision.

8 Standards for Mathematical Practice

7) Look for and make use of structure.

8 Standards for Mathematical Practice

8) Look for and express regularity in repeated reasoning.

Standards for Mathematical Content

There are content standards at each K-8 grade level.
The "Higher Mathematics" (a.k.a. high school) content standards are grouped into 6 categories.

6 Conceptual Categories for higher mathematics

1. Number and Quantity
2. Algebra
3. Functions
4. Modeling
5. Geometry
6. Statistics and Probability

"The higher mathematics standards specify the mathematics that all students should study in order to be college and career ready…"

"The higher mathematics standards specify the mathematics that all students should study in order to be college and career ready. Additional mathematics that students should learn in preparation for advanced courses, such as calculus, advanced statistics, or discrete mathematics, is indicated by a plus symbol (+). All standards without a (+) symbol should be in the common mathematics curriculum for all college and career ready students. Standards with a (+) symbol may also appear in courses intended for all students."

Higher Mathematics Content Standards

The CA CCSSM suggests two possible pathways to include the higher mathematics content standards: a Traditional Pathway (Algebra I, Geometry, and Algebra II) and an Integrated Pathway (Mathematics I, II, and III).

CCSS Algebra I includes some non-traditional topics

Linear, quadratic, and exponential functions, including arithmetic and geometric sequences as functions, function notation, and fitting functions to data

CCSS Algebra I includes some non-traditional topics

Statistics, including assessing the fit of a function by plotting and analyzing residuals; interpreting the slope, intercept, and correlation coefficient of a linear model in context.

CCSS Geometry includes some non-traditional topics

Transformational geometry: congruence defined in terms of rigid motion; similarity defined in terms of dilations and rigid motions.

CCSS Geometry includes some non-traditional topics

Trigonometry: trig ratios, special angles, derivation of the equation of a parabola given a focus and directrix.

CCSS Geometry includes some non-traditional topics

Probability: sample spaces, independent events, conditional probability, permutations and combinations; analyzing decisions and strategies using probability

CCSS Algebra II includes some non-traditional topics

Trigonometry: 6 trig functions of real numbers; modeling periodic phenomena, proof and use of the identity $\sin^2\theta + \cos^2\theta = 1$

CCSS Algebra II includes some non-traditional topics

Statistics: normal distributions, random samples, estimating population parameters, simulations, using probability to make decisions

Examples of (+) standards in traditional intermediate Algebra

(+) Find the conjugate of a complex number; use conjugates to find moduli and quotients of complex numbers.

(+) Extend polynomial identities to the complex numbers. *For example, rewrite* $x^2 + 4$ *as* $(x + 2i)(x – 2i)$.

(+) Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials.

Examples of (+) standards in traditional intermediate Algebra

(+) Know and apply the Binomial Theorem for the expansion of $(x + y)^n$ in powers of x and y for a positive integer n, where x and y are any numbers, with coefficients determined for example by Pascal's Triangle

Examples of (+) standards in traditional intermediate Algebra

(+) Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions.

Examples of (+) standards in traditional intermediate Algebra

(+) Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior.

Examples of (+) standards in traditional intermediate Algebra

(+) Verify by composition that one function is the inverse of another.

(+) Read values of an inverse function from a graph or a table, given that the function has an inverse.

Examples of (+) standards in traditional intermediate Algebra

(+) Produce an invertible function from a non-invertible function by restricting the domain.

(+) Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents.

More (+) standards

Geometry of complex numbers (3 standards)
Vectors (5 standards)
Matrices (9 standards)
Trig (6 standards)
Geometry (3 standards)
Probability/stats (9 standards)

Do CCCs need to align with CCSSM?

The Student Success Task Force recommends it:

"Aligning K-12 and community colleges standards for college and career readiness is a long-term goal that will require a significant investment of time and energy that the Task Force believes will pay off by streamlining student transition to college and reducing the academic deficiencies of entering students…

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"Recommendation 1.1: Community Colleges will collaborate with K-12 education to jointly develop new common standards for college and career readiness that are aligned with high school exit standards."

Do CCCs need to align with CCSSM? (continued)

The University of California expects it:

The UC Board of Admissions and Relations with Schools (BOARS) wrote in July 2013 that "… the basic mathematics of the CCSSM can appropriately be used to define the minimal level of mathematical competence that all incoming UC students should demonstrate."

BOARS clarified (December 2013) that "… going forward, all students must complete the basic mathematics defined by the college-ready standards of the Common Core State Standards for Mathematics (CCSSM) prior to enrolling in a UC-transferable college mathematics or statistics course."

What does the UC mean by CCSSM "alignment"?

"Much of the longstanding discussion surrounding what foundational mathematics is necessary for college-level mathematics focuses on algebra. But it is important to note that algebra is only one of several topics identified in the CCSSM. Also specified are number and quantity, functions, modeling, geometry, and statistics and probability." (12/2013)

What does the UC mean by CCSSM "alignment"? (continued)

"Specifying that transferable courses must have at least Intermediate Algebra as a prerequisite is not fully consistent with the use of the basic mathematics of the CCSSM as a measure of college readiness …."

(7/13)

What does the UC mean by CCSSM "alignment"? (continued)

"Specifying that transferable courses must have at least Intermediate Algebra as a prerequisite is not fully consistent with the use of the basic mathematics of the CCSSM as a measure of college readiness in that most existing Intermediate Algebra courses contain topics that are identified in the CCSSM as part of the (+) standards." (7/13)

What does the UC mean by CCSSM "alignment"? (continued)

"Because current course offerings of Intermediate Algebra include material identified in the CCSSM as "additional mathematics that students should learn in order to take advanced courses such as calculus, advanced statistics, or discrete mathematics," it will not be appropriate in the future to use traditional Intermediate Algebra (i.e., Intermediate Algebras as defined prior to CCSSM implementation) as the primary standard for demonstrating the minimal level of mathematical competence that BOARS seeks in students admitted to UC." (7/13)

What does the UC mean by CCSSM "alignment"? (continued)

"Requiring that all prospective transfer students pass the current version of Intermediate Algebra would be asking more of them than UC will ask of students entering as freshmen who have completed CCSSM-aligned high school math courses…"

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What does the UC mean by CCSSM "alignment"? (continued)

"Requiring that all prospective transfer students pass the current version of Intermediate Algebra would be asking more of them than UC will ask of students entering as freshmen who have completed CCSSM-aligned high school math courses. As such, BOARS expects that the Transferable Course Agreement Guidelines will be rewritten to clarify that the prerequisite mathematics for transferable courses should align with the college-ready content standards of the CCSSM." (7/13)

What does the UC mean by CCSSM "alignment"? (continued)

"BOARS acknowledges that the continued use of Intermediate Algebra as the prerequisite for UC-transferable courses is problematic. Such courses traditionally cover more advanced topics than are included in the basic college-ready CCSSM standards. …"

(12/2013)

What does the UC mean by CCSSM "alignment"? (continued)

"BOARS acknowledges that the continued use of Intermediate Algebra as the prerequisite for UC-transferable courses is problematic. Such courses traditionally cover more advanced topics than are included in the basic college-ready CCSSM standards. Thus, BOARS's statement closes with the expectation that future UC-transferable courses will have prerequisites that align with the Common Core, not prerequisites that have a particular name." (12/2013)

What does the UC mean by CCSSM "alignment"? (continued)

"BOARS recognizes that this is a period of transition in mathematics instruction, moving from traditional course sequences to new courses and sequences. Within the CCSSM, there are multiple pathways to meet the college-ready standards, and BOARS encourages the development of such new approaches within the California Community Colleges…" (12/2013)

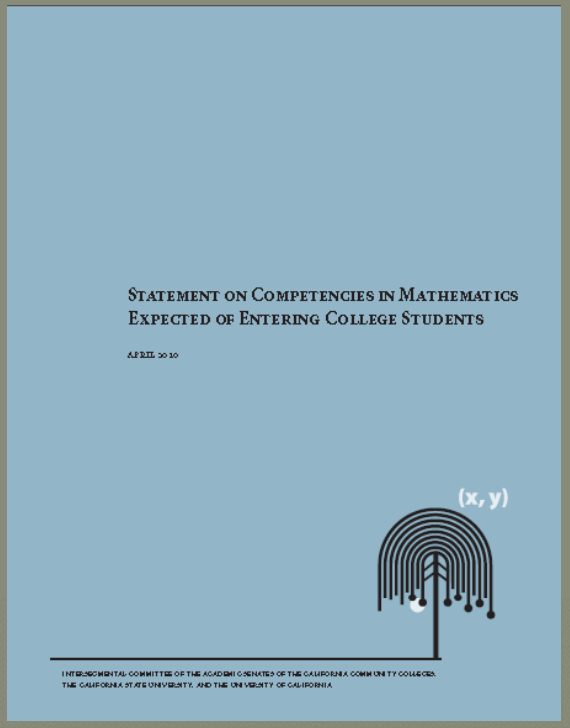
What does the UC mean by CCSSM "alignment"? (continued)

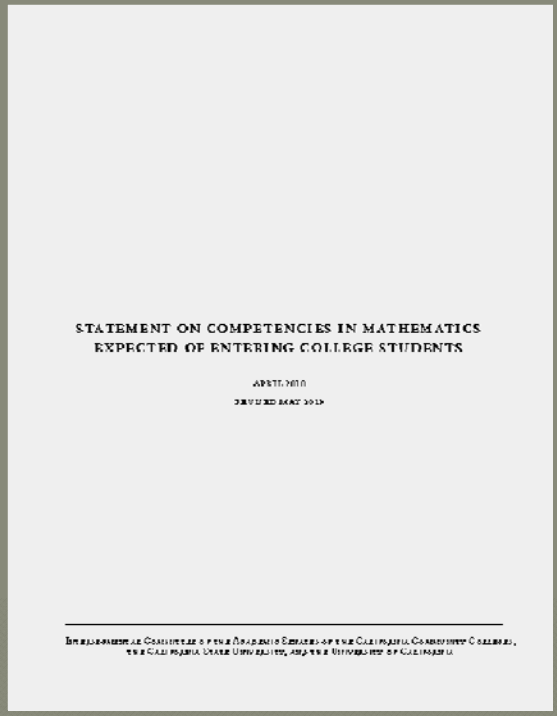
"BOARS recognizes that this is a period of transition in mathematics instruction, moving from traditional course sequences to new courses and sequences. Within the CCSSM, there are multiple pathways to meet the college-ready standards, and BOARS encourages the development of such new approaches within the California Community Colleges. The key is to ensure that students have met the standards of the Common Core State Standards for Mathematics, not that they have completed a specific course." (12/2013)

What does the UC mean by CCSSM "alignment"? (continued)

According to the July UC BOARS statement: "The most recent version of the ICAS mathematical competency statement makes clear the close alignment between it and the CCSSM. Both define the mathematics that all students should study in order to be college ready." [Emphasis mine]

The Intersegmental Committee of the Academic Senates: "The goal of this Statement on Competencies in Mathematics Expected of Entering College Students is to provide a clear and coherent message about the mathematics that students need to know and to be able to do to be successful in college. "





What ICAS does NOT consider necessary for all students (but is in CCSSM):

Right triangle trigonometry; transformational geometry, including dilations. (ICAS lists only as "desirable")

What ICAS does NOT consider necessary for all students (but is in CCSSM):

Solutions to systems of equations and their geometrical interpretation; solutions to quadratic equations, both algebraic and graphical; complex numbers and their arithmetic; the correspondence between roots and factors of polynomials; rational expressions; the binomial theorem. (ICAS lists only for STEM)

What ICAS does NOT consider necessary for all students (but is in CCSSM):

Trigonometric functions of real variables, their graphs, properties including periodicity, and applications to right triangle trigonometry; basic trigonometric identities. (ICAS lists only for STEM)

What ICAS does NOT consider necessary for all students (but is in CCSSM):

Two- and three-dimensional coordinate geometry; locus problems. (ICAS lists only for STEM)

What ICAS does NOT consider necessary for all students (but is in CCSSM):

Distributions as models; the Normal Distribution; fitting data with curves; correlation, regression; sampling, graphical displays of data. (ICAS lists only for STEM)

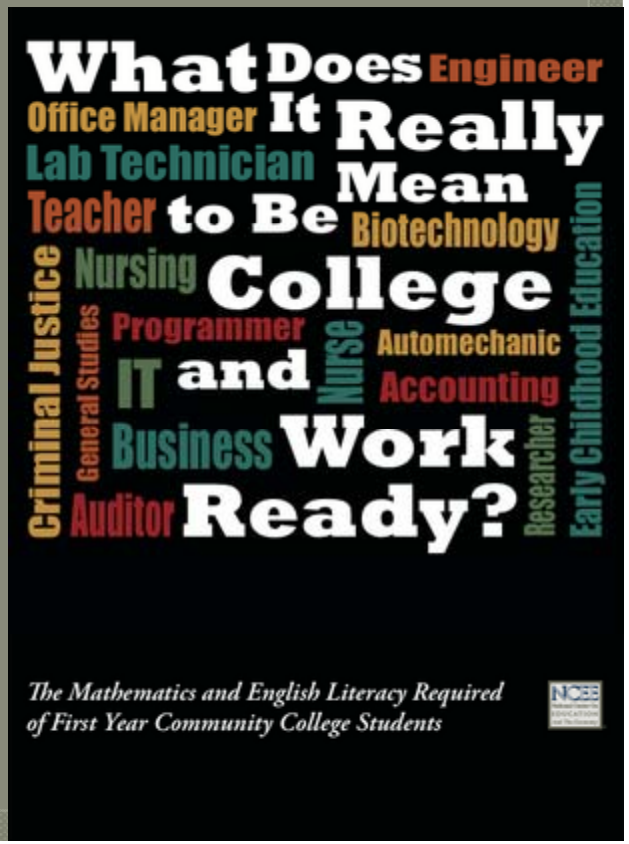
What ICAS does NOT consider necessary for all students (but is in CCSSM):

Conic sections: representations as plane sections of a cone; focus-directrix properties; reflective properties. (ICAS lists only for STEM)

Aside: The National Center on Education and the Economy (May 2013)

"Mastery of Algebra II is widely thought to be a prerequisite for success in college and careers. Our research shows that that is not so… Based on our data, one cannot make the case that high school graduates must be proficient in Algebra II to be ready for college and careers."

http://www.ncee.org/college-and-work-ready/



What does the UC mean by CCSSM "alignment"? (continued)

The ICAS mathematical competency statement begins with "Part 1: Dispositions of well-prepared students toward mathematics."

- A view that mathematics makes sense—students should perceive mathematics as a way of understanding, not as a sequence of algorithms to be memorized and applied.

- An ease in using their mathematical knowledge to solve unfamiliar problems in both concrete and abstract situations—students should be able to find patterns, make conjectures, and test those conjectures; they should recognize that abstraction and generalization are important sources of the power of mathematics; they should understand that mathematical structures are useful as representations of phenomena in the physical world; they should consistently verify that their solutions to problems are reasonable.

- A willingness to work on mathematical problems requiring time and thought, problems that are not solved by merely mimicking examples that have already been seen—students should have enough genuine success in solving such problems to be confident, and thus to be tenacious, in their approach to new ones.

•A readiness to discuss the mathematical ideas involved in a problem with other students and to write clearly and coherently about mathematical topics—students should be able to communicate their understanding of mathematics with peers and teachers using both formal and natural languages correctly and effectively.

- An acceptance of responsibility for their own learning—students should realize that their minds are their most important mathematical resource, and that teachers and other students can help them to learn but can't learn for them.

- The understanding that assertions require justification based on persuasive arguments, and an ability to supply appropriate justifications—students should habitually ask "Why?" and should have a familiarity with reasoning at a variety of levels of formality, ranging from concrete examples through informal arguments using words and pictures to precise structured presentations of convincing arguments.

- While proficiency in the use of technology is not a substitute for mathematical competency, students should be familiar with and confident in the use of computational devices and software to manage and display data, to explore functions, and to formulate and investigate mathematical conjectures.

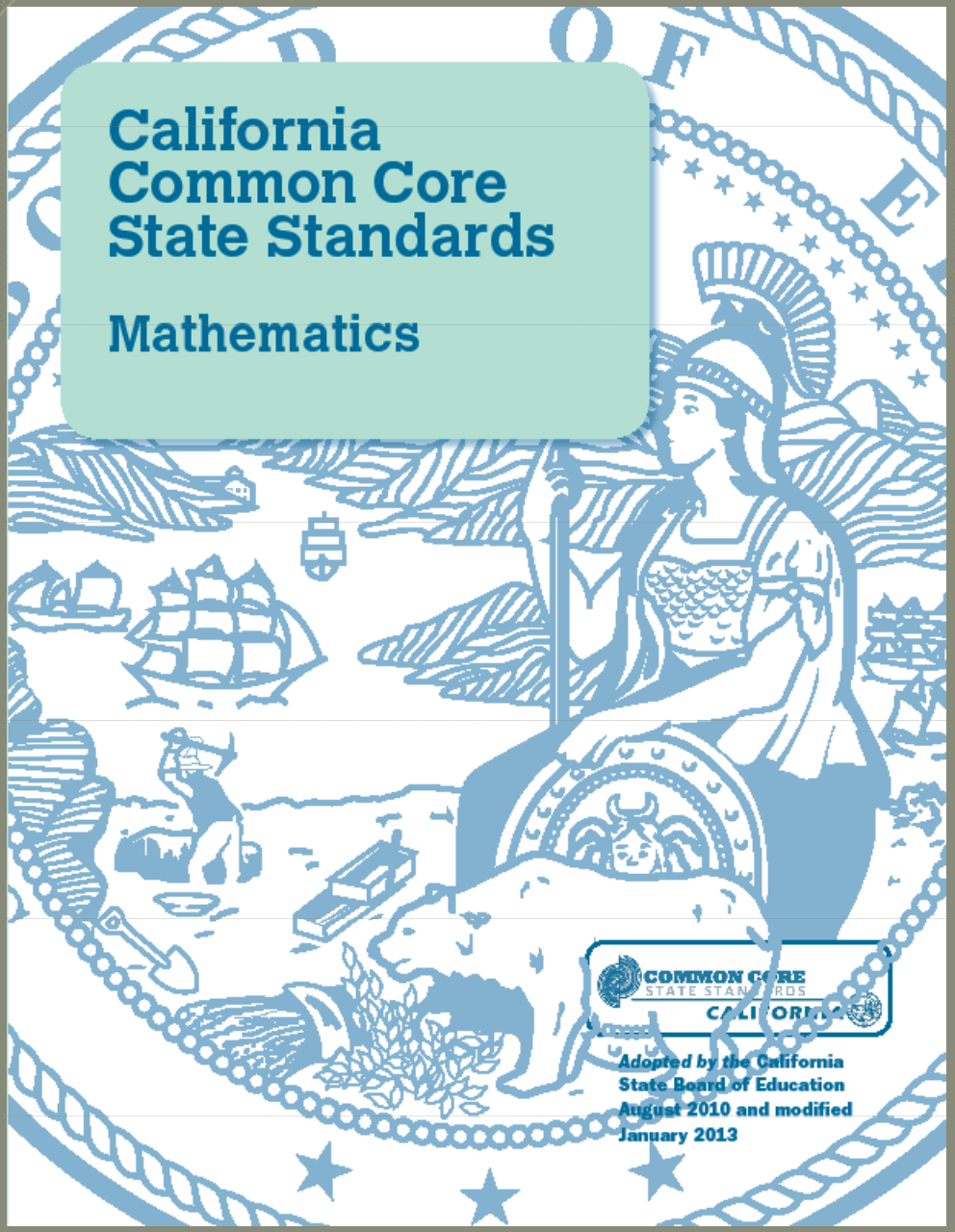
- A perception of mathematics as a unified field of study—students should see interconnections among various areas of mathematics, which are often perceived as distinct.

CCSSM Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

What does the UC mean by CCSSM "alignment"? (continued)

The ICAS math Dispositions and the CCSSM standards for Mathematical Practice are consistent…is that sufficient for "close alignment"?



The entire CCSSM document and ancillaries are available for free download from the CA Dept of Ed website:
http://www.cde.ca.gov/re/cc/

The approved draft of the CA Math Framework is also available online:
http://www.cde.ca.gov/ci/ma/cf/

Thank you!

Bruce Yoshiwara

(A Google search on my name should find my homepage, and from there links to handouts/information for faculty, including this file and links to related resources.)

"The eight Standards for Mathematical Practice (MP) describe the attributes of mathematically proficient students and expertise that mathematics educators at all levels should seek to develop in their students. Mathematical practices provide a vehicle through which students engage with and learn mathematics. As students move from elementary school through high school, mathematical practices are integrated in the tasks as students engage in doing mathematics and master new and more advanced mathematical ideas and understandings." (CA Framework)

More (+) standards

(+) Represent complex numbers on the complex plane in rectangular and polar form (including real and imaginary numbers), and explain why the rectangular and polar forms of a given complex number represent the same number.

More (+) standards

(+) Represent addition, subtraction, multiplication, and conjugation of complex numbers geometrically on the complex plane; use properties of this representation for computation. For example, $(-1 + \sqrt{3}\, i)^3 = 8$ because $(-1 + \sqrt{3}\, i)$ has modulus 2 and argument 120°.

(+) Calculate the distance between numbers in the complex plane as the modulus of the difference, and the midpoint of a segment as the average of the numbers at its endpoints.

More (+) standards

Represent and model with vector quantities (3 standards).

Perform operations on vectors (2 standards).

More (+) standards

(+) Use matrices to represent and manipulate data, e.g., to represent payoffs or incidence relationships in a network.

(+) Multiply matrices by scalars to produce new matrices, e.g., as when all of the payoffs in a game are doubled.

More (+) standards

(+) Add, subtract, and multiply matrices of appropriate dimensions.

(+) Understand that, unlike multiplication of numbers, matrix multiplication for square matrices is not a commutative operation, but still satisfies the associative and distributive properties.

More (+) standards

(+) Understand that the zero and identity matrices play a role in matrix addition and multiplication similar to the role of 0 and 1 in the real numbers. The determinant of a square matrix is nonzero if and only if the matrix has a multiplicative inverse.

(+) Multiply a vector (regarded as a matrix with one column) by a matrix of suitable dimensions to produce another vector. Work with matrices as transformations of vectors.

(+) Work with 2 × 2 matrices as transformations of the plane, and interpret the absolute value of the determinant in terms of area.

More (+) standards

(+) Represent a system of linear equations as a single matrix equation in a vector variable.

(+) Find the inverse of a matrix if it exists and use it to solve systems of linear equations (using technology for matrices of dimension 3×3 or greater).

More (+) standards

(+) Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed.

(+) Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context.

More (+) standards

(+) Prove the addition and subtraction formulas for sine, cosine, and tangent and use them to solve problems.

(+) Derive the formula A = 1/2 ab sin(C) for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side.

More (+) standards

(+) Prove the Laws of Sines and Cosines and use them to solve problems.

(+) Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems, resultant forces).

More (+) standards

(+) Construct a tangent line from a point outside a given circle to the circle.

(+) Derive the equations of ellipses and hyperbolas given the foci, using the fact that the sum or difference of distances from the foci is constant.

(+) Give an informal argument using Cavalieri's principle for the formulas for the volume of a sphere and other solid figures.

More (+) standards

(+) Apply the general Multiplication Rule in a uniform probability model, P(A and B) = P(A)P(B|A) = P(B)P(A|B), and interpret the answer in terms of the model.

(+) Use permutations and combinations to compute probabilities of compound events and solve problems.

More (+) standards

Calculate expected values and use them to solve problems (4 standards)

Use probability to evaluate outcomes of decisions (3 standards)